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The perturbative pion-photon transition form factors with transverse momentum corrections

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Abstract

We perform a perturbative QCD analysis of the quark transverse momentum effect on the pion-photon transition form factors $F_{\pi\gamma}$ and $F_{\pi\gamma^*}$ in the standard light-cone formalism, with two phenomenological models of wavefunction as the input of the non-perturbative aspect of the pion. We point out that the transverse momentum dependence in both the numerator and the denominator of the hard scattering amplitude is of the same importance and should be considered consistently. It is shown that after taking into account the quark transverse momentum corrections, the results obtained from different model wavefunctions are consistent with the available experimental data at finite Q^2.

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I. Introduction

The pion-photon transition form factor $F_{\pi\gamma}(Q^2)$ is a simple example for the perturbative analysis to exclusive processes and was first analysed by Lepage and Brodsky [1]. They predicted $F_{\pi\gamma}(Q^2)$ by neglecting $k_\perp$ relative to $q_\perp$,

$$F_{\pi\gamma}(Q^2) = \frac{2}{\sqrt{3}Q^2}\int \frac{[\mathrm{d}x]}{x_1 x_2}\phi_\pi(x)\left[1 + \mathrm{O}\left(\alpha_s, \frac{m^2}{Q^2}\right)\right], \tag{1}$$

and $Q^2 F_{\pi\gamma}(Q^2)$ would be essentially constant as $Q^2 \to \infty$. This approximation would be valid if the wavefunction is peaked at low $k_\perp$ ($k_\perp$ is the transverse momentum of quark) so that $x_1 x_2 Q^2$ in the hard scattering amplitude dominates the denominator. However, at the end-point region $x_i \to 0, 1$ and $Q^2 \sim$ a few GeV^2 the wavefunction does not guarantee the $k_\perp$ negligible. One should take into account $k_\perp$ corrections from both the hard scattering amplitude and the wavefunction.

Recently, Refs. [2, 3] calculated the π-γ transition form factor within the covariant hard scattering approach including transverse momentum effects and Sudakov corrections [4] by neglecting the quark masses, the mass of the pion meson and the $k_\perp$-dependence in the numerator of T_H. Their results show that Sudakov suppression in the form factor $F_{\pi\gamma}(Q^2)$ is less important than in other exclusive channels and the Chernyak-Zhitnitsky(CZ) wavefunction should be discarded by fitting the experimental data. However, as we know, the $k_\perp$-dependence of the wavefunction in Ref. [2] is the same in the different models and it may be difficult to draw a conclusion which excludes the CZ wavefunction. We will re-examine this problem in the present paper.

The light-cone formalism provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and for the application of perturbative QCD (pQCD) to exclusive processes [5, 6]. In this formalism, the hadronic wavefunction which describes the hadronic composite state at a particular

τ is expressed in terms of a series of light-cone wavefunctions in Fock-state basis,

$$|\pi\rangle = \sum |q\bar{q}\rangle \psi_{q\bar{q}} + \sum |q\bar{q}g\rangle \psi_{q\bar{q}g} + \cdots, \tag{2}$$

and the temporal evolution of the state is generated by the light-cone Hamiltonian $H_{LC} = P^- = P^0 - P^3$. Furthermore the vacuum state in the light-cone Fock basis is an exact eigen-state of the full Hamiltonian H_{LC}. Thus all bare quanta in a hadronic Fock state are part of the hadron (This point is very different from that in the equal-t perturbative theory in which the quantization is performed at a given time t). Light-cone pQCD is very convenient for light-cone dominated processes. For the detail quantization rules we refer to literatures [1, 5, 6, 7]. The more important point for practical calculation is that the contributions coming from higher Fock states are suppressed by $1/Q^n$, therefore we can employ only the valence state to the leading order for large Q^2. In this paper, we analyze the quark transverse momentum effects on the pion-photon transition form factors $F_{\pi\gamma}$ and $F_{\pi\gamma^*}$ at finite Q^2 in the standard light-cone formalism, with two phenomenological models of wavefunction as the input of the non-perturbative aspect of the pion. We demonstrate that the pQCD predictions with the different models of wavefunction are consistent with the available experimental data by taking into account the quark transverse momentum.

II. The pion-photon transition form factors $F_{\pi\gamma}$ and $F_{\pi\gamma^*}$

The π-γ transition form factor $F_{\pi\gamma}$ is defined from the $\pi^0\gamma\gamma^*$ vertex in the amplitude of $e\pi \to e\gamma$,

$$\Gamma_\mu = -ie^2 F_{\pi\gamma} \epsilon_{\mu\nu\alpha\beta} p_\pi^\mu \epsilon^\alpha q^\beta, \tag{3}$$

where p_π and q are the momenta of the incident pion and the virtual photon respectively, and ϵ is the polarization vector of the final (on-shell) photon. We adopt the

standard momentum assignment at the "infinite-momentum" frame [1]

$$\begin{aligned} p_\pi &= (p^+, p^-, p_\perp) = (1, 0, 0_\perp), \\ q &= (0, q_\perp^2, q_\perp), \end{aligned} \tag{4}$$

where p^+ is arbitrary. For simplicity we choose $p^+ = 1$, and we have $q^2 = -q_\perp^2 = -Q^2$. Then the $F_{\pi\gamma}$ is given by

$$F_{\pi\gamma}(Q^2) = \frac{\Gamma^+}{-ie(\epsilon_\perp \times q_\perp)}, \tag{5}$$

where $\epsilon = (0, 0, \epsilon_\perp)$, $\epsilon_\perp \cdot q_\perp = 0$ is chosen and $\epsilon_\perp \times q_\perp = \epsilon_{\perp 1} q_{\perp 2} + \epsilon_{\perp 2} q_{\perp 1}$. Since the contributions coming from higher Fock states are suppressed, we take into account only the conventional lowest Fock state of pion meson,

$$\psi_\pi = \frac{\delta_b^a}{\sqrt{n_c}} \frac{1}{\sqrt{2}} \left[\frac{u_\uparrow \bar{u}_\downarrow - u_\downarrow \bar{u}_\uparrow}{\sqrt{2}} - \frac{d_\uparrow \bar{d}_\downarrow - d_\downarrow \bar{d}_\uparrow}{\sqrt{2}} \right] \frac{\psi(x_i, k_\perp)}{\sqrt{x_1 x_2}}. \tag{6}$$

The leading-order contribution to $F_{\pi\gamma}$ is calculated from Fig. 1 in light-cone pQCD [1],

$$\begin{aligned} F_{\pi\gamma}(Q^2) = & \frac{\sqrt{n_c}(e_u^2 - e_d^2)}{i(\epsilon_\perp \times q_\perp)} \int_0^1 [\mathrm{d}x] \int_0^\infty \frac{\mathrm{d}^2 k_\perp}{16\pi^3} \psi(x_i, k_\perp) \\ & \left[\frac{\bar{v}_\downarrow(x_2, -k_\perp)}{\sqrt{x_2}} \not{\epsilon} \frac{u_\uparrow(x_1, k_\perp + q_\perp)}{\sqrt{x_1}} \frac{\bar{u}_\uparrow(x_1, k_\perp + q_\perp)}{\sqrt{x_1}} \gamma^+ \frac{u_\uparrow(x_1, k_\perp)}{\sqrt{x_1}} \frac{1}{D} + (1 \leftrightarrow 2) \right], \end{aligned} \tag{7}$$

where $[\mathrm{d}x] = \mathrm{d}x_1 \mathrm{d}x_2 \delta(1 - x_1 - x_2)$, $e_{u,d}$ are the quark charges in unites of e, and D is the "energy-denominator",

$$D = q_\perp^2 - \frac{(k_\perp + q_\perp)^2 + m^2}{x_1} - \frac{k_\perp^2 + m^2}{x_2}. \tag{8}$$

The quark masses relative to Q^2 can be neglected since they are the current quark masses in pQCD calculation. Thus Eq. (7) becomes

$$F_{\pi\gamma}(Q^2) = 2\sqrt{n_c}(e_u^2 - e_d^2) \int_0^1 [\mathrm{d}x] \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3} \psi(x_i, k_\perp) \times T_H(x_1, x_2, k_\perp), \tag{9}$$

where

$$T_H(x_1, x_2, k_\perp) = \frac{q_\perp \cdot (x_2 q_\perp + k_\perp)}{q_\perp^2 (x_2 q_\perp + k_\perp)^2} + (1 \leftrightarrow 2). \tag{10}$$

The leading behavior of T_H (at large Q^2) is obtained by neglecting $k_\perp$ relative to $x_i q_\perp$ [6],

$$T_H^{LO}(x_1, x_2, k_\perp) = \frac{1}{x_1 x_2 Q^2}. \tag{11}$$

Thus Eq. (9) become Eq. (1) to the leading order. The higher twist corrections to T_H^{LO} take the forms of $\left(\frac{k_\perp}{x_i Q}\right)^n$. Eq. (10) tells us that there are two factors to contribute for the $k_\perp$-dependence. One is from the pQCD hard scattering amplitude $T_H(x_i, Q, k_\perp)$, and another one is from the non-perturbative wavefunction $\psi(x_i, k_\perp)$. Although one hopes that the end-point behavior of the wavefunction can guarantee the reliability of neglecting these higher twist corrections and can suppress the end-point singularity, these corrections may substantially modify the predictions for $F_{\pi\gamma}$ at the momentum transfer Q of a few GeV, especially for the wavefunction with a milder suppression factor in the end-point region. It should be emphasized that the $k_\perp$-dependence in the numerator and the denominator of T_H is of the same importance. Thus one can not simply ignore the $k_\perp$ term in the numerator of T_H and Eq. (10) gives the complete expression in the leading order.

The π-γ^* transition form factor $F_{\pi\gamma^*}$ is extracted from the $\pi^0\gamma^*\gamma^*$ vertex in the two-photon physics. Once again, we employ the standard momentum assignment at the "infinite-momentum" frame

$$\begin{aligned} p_\pi &= (p^+, p^-, p_\perp) = (1, 0, 0_\perp), \\ q &= (0, q_\perp^2 - Q'^2, q_\perp), \\ q' &= (1, q_\perp^2 - Q'^2, q_\perp), \end{aligned} \tag{12}$$

where q and q' are the momenta of the two photons respectively, and $q^2 = -q_\perp^2 = -Q^2$, $q'^2 = -Q'^2$. $F_{\pi\gamma^*}$ may be calculated from Fig. 1 by substituting a virtual photon γ^* for the on-shell photon γ, which gives

$$\begin{aligned} F_{\pi\gamma}(Q^2, Q'^2) &= 2\sqrt{n_c}(e_u^2 - e_d^2)\int_0^1 [\mathrm{d}x] \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3}\psi(x_i, k_\perp) \\ & \left[\frac{q_\perp \cdot (x_2 q_\perp + k_\perp)}{q_\perp^2\left[(x_2 q_\perp + k_\perp)^2 + x_1 x_2 q_\perp'^2\right]} + (1 \leftrightarrow 2)\right]. \end{aligned} \tag{13}$$

The leading order behavior of $F_{\pi\gamma^*}$ can be obtained from Eq. (13) by neglecting $k_\perp$ relative to $x_i q_\perp$ [1],

$$F_{\pi\gamma^*}(Q^2, Q'^2) = 2\sqrt{n_c}(e_u^2 - e_d^2)\int_0^1 [\mathrm{d}x]\phi_\pi(x)\left[\frac{1}{x_2 Q^2 + x_1 Q'^2} + (1 \leftrightarrow 2)\right]. \tag{14}$$

Similar to the $F_{\pi\gamma}$, Eq. (13) may substantially modifies the predictions obtained from Eq. (14).

III. Numerical calculations

In order to see the transverse momentum corrections, we employ two models of wavefunction: (a) the Brodsky-Huang-Lepage (BHL) wavefunction [5]

$$\psi^{BHL}(x, k_\perp) = A\mathrm{exp}\left[-\frac{\mathrm{k}_\perp^2 + \mathrm{m}^2}{8\beta^2\mathrm{x}(1-\mathrm{x})}\right], \tag{15}$$

where $A = 32\mathrm{GeV}^{-1}$, $\beta = 0.385$ GeV and $m = 289$ MeV [8]; (b) the CZ-like wavefunction [9]

$$\psi^{CZ}(x, k_\perp) = A(1-2x)^2\mathrm{exp}\left[-\frac{\mathrm{k}_\perp^2 + \mathrm{m}^2}{8\beta^2\mathrm{x}(1-\mathrm{x})}\right], \tag{16}$$

where $A = 136\mathrm{GeV}^{-1}$, $\beta = 0.455$ GeV and $m = 342$ MeV [8]. These models express that the Fock state wavefunction $\psi(x_i, k_\perp)$ in the infinite momentum frame depends on the off-shell energy variable $\varepsilon = \sum_i^n \left(\frac{k_{\perp i}^2 + m_i^2}{x_i}\right)$, which was pointed out in Ref. [5].

Substituting the models (15) and (16) into Eqs. (9), (10) and (13), one can get the transverse momentum corrections to the pion-photon transition form factor. The results of $F_{\pi\gamma}$ calculated with ψ^{BHL} and ψ^{CZ} are plotted in Fig. 2. The dashed curves are calculated from the hard scattering amplitude T_H^{LO} in the leading order without the transverse momentum corrections (see Eq. (1)), and the constant predictions with the different wavefunctions can not describe the experimental data at momentum transfer of a few GeV^2 explicitly. The solid curves are obtained from the complete expression of T_H (see Eq. (10)) with the transverse momentum corrections. As expected, the higher twist correction are suppressed by $1/Q^2$ and the prediction approaches to a constant which depends on the wavefunction at large Q region. The perturbative predictions are smaller than the experimental data, especially for Q^2 of $1 \sim 3$ GeV^2, which supports the suggestion that the higher order effects should provide some contributions at experimental accessible momentum transfer and become more important with Q^2 decreasing. Although the asymptotic behaviors of $F_{\pi\gamma}$ predicted from the BHL model and CZ-like model of wavefunction are quit different, their predictions at finite Q^2 obtained with transverse momentum corrections are consistent with the experimental data. The reason is as following: There are two factors to affect the prediction with the CZ-like wavefunction. First, the CZ-like model emphasizes the end-point region in a strong way, which enhance its prediction of $F_{\pi\gamma}$. Second, the transverse momentum corrections become more important in the end-point region, which make its prediction decrease. Combining these two factors, the CZ-like model gives a very similar prediction as the BHL model in the finite momentum transfer region. Thus, neither of the two models of wavefunction can be excluded by the available data of this exclusive process.

The results of $F_{\pi\gamma^*}$ calculated with ψ^{BHL} and ψ^{CZ} are plotted in Fig. 3. Once again, the higher twist corrections are suppressed by $1/Q^2$ and provide more contributions as Q^2 decreasing. The predictions of the two models are not different dramatically, no matter the transverse momentum corrections are taken into account or not, since the energy scale Q' coming from the other virtual photon makes the hard scattering amplitude is not as singular as that in the case of $F_{\pi\gamma}$. It is also difficult to exclude one of the two models of wavefunction basing on $F_{\pi\gamma^*}$. At present, the lack of experiment data make the examination of higher twist effects in $F_{\pi\gamma^*}$ more complex than that in $F_{\pi\gamma}$. But the future high-luminosity e^+e^- colliders in the "τ-charm factory" or "B factory" will make this examination feasible.

III. Summary

In summary, we emphasize again that the light-cone perturbative QCD is a natural framework to calculate the large-momentum-transfer exclusive processes. It is reasonable to get the higher twist corrections by taking into account the quark transverse momentum dependence. As $Q^2 \to \infty$, these corrections become negligible. After taking into account the transverse momentum dependence, pQCD may give correct prediction for the pion-photon transition form factor which is consistent with the experimental data. The transverse-momentum-dependence in both the numerator and the denominator of the hard scattering amplitude T_H is of the same importance and should be considered consistently. Neither the BHL model nor the CZ-like model, the two typical models of wavefunctions, can be excluded by the available data of the pion-photon transition form factors. The future "τ-charm factory" as well as "B factory" will provide the opportunity to examine the higher twist effects in the perturbative

calculation of $F_{\pi\gamma^*}$ and to test the validity of the perturbative analysis.

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References

[1] G.P. Lepage and S.J. Brodsky, Phys. Rev. **D 22**, 2157 (1980), *ibid.* **24**, 1808 (1981).

[2] R. Jakob, P. Kroll, and M. Raulfs, preprint hep-ph 9410304 (1994); P. Kroll, preprint hep-ph 9504394 (1995).

[3] S. Ong, Phys. Rev. **D 52**, 3111 (1995).

[4] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989); H.N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992); F.G. Cao, T. Huang, and C.W. Luo, Phys. Rev. **D 52**, 5358 (1995).

[5] S.J. Brodsky, T.Huang, and G.P. Lepage, in *Particles and Fields-2*, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), p. 143. T. Huang, Proceedings of XX-th International Conference on High Energy Physics, Madison, Wisconsin, 1980.

[6] S.J. Brodsky and G.P. Lepage, In *Perturbative Quantum Chromodynamics*, edited by A.H. Mueller (World Scientific, Singapore 1989), p. 93.

[7] See, e.g., J.B. Kogut and D.E. Soper, Phys. Rev. **D 1**, 2901 (1970); J.B. Bjorken, J.B. Kogut, and D.E. Soper, *ibid.* **3**, 1328 (1971); S.J. Brodsky, R. Roskies, and R. Suaya, *ibid.* **8**, 4574 (1973).

[8] T. Huang, B.-Q. Ma, and Q.-X. Shen, Phys. Rew. **D 49**, 1490 (1994).

[9] T. Huang and Q.-X. Shen, Z. Phys. **C 50**, 139 (1991).

[10] CELLO Collab., H.-J. Behrend *et al.*, Z. Phys. **C 49**, 401 (1991), there is a factor of $4\pi\alpha$ for the definition of form factors between our analysis and this reference.

[11] CLEO Collab., V. Savinov *et al.*, hep-ex/9507005 (1995).

Figure Captions

Fig. 1 The lowest order diagrams contributing to $F_{\pi\gamma}$ in light-cone pQCD.

Fig. 2 The π-γ transition form factor. The solid curves are obtained by taking into account the $k_\perp$-dependence, while the dashed curves are results without $k_\perp$-dependence. In both of the cases, the thick curves are calculated from the BHL wave function and the thin curves are for the CZ-like wavefunction. The data are taken from Refs. [10, 11].

Fig. 3 The π-γ^* form factor at $Q'^2 = 2$ GeV^2. The explanation of the curves is similar to Fig. 2.

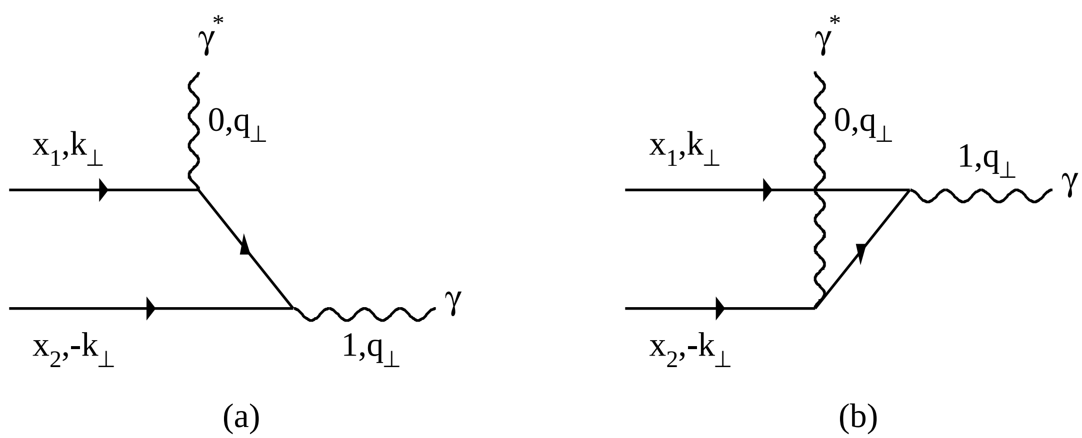
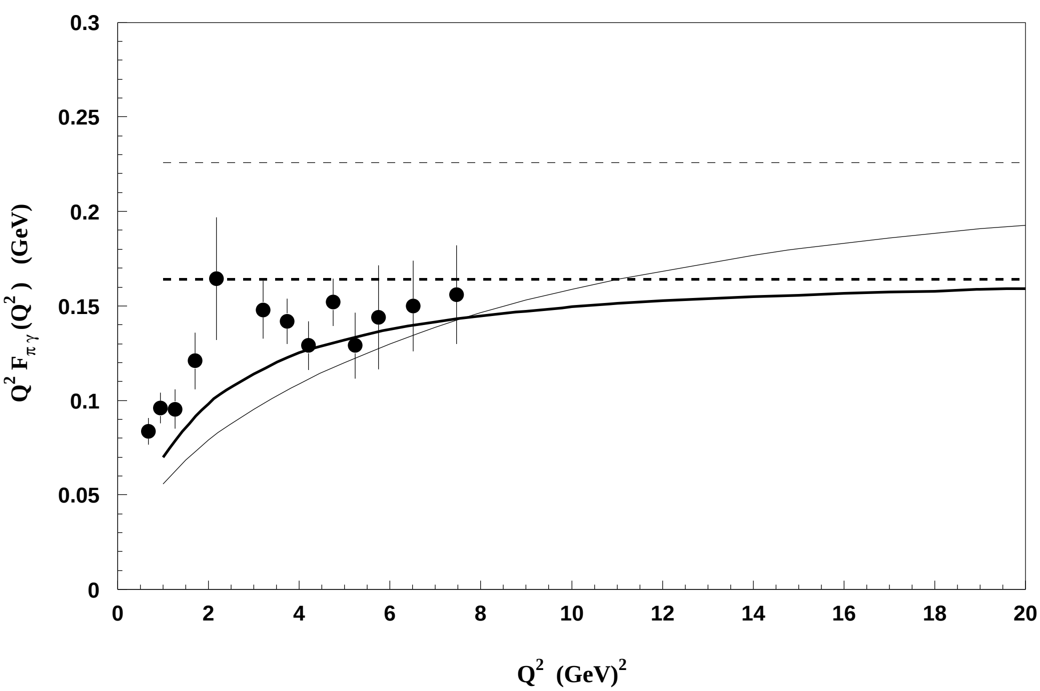
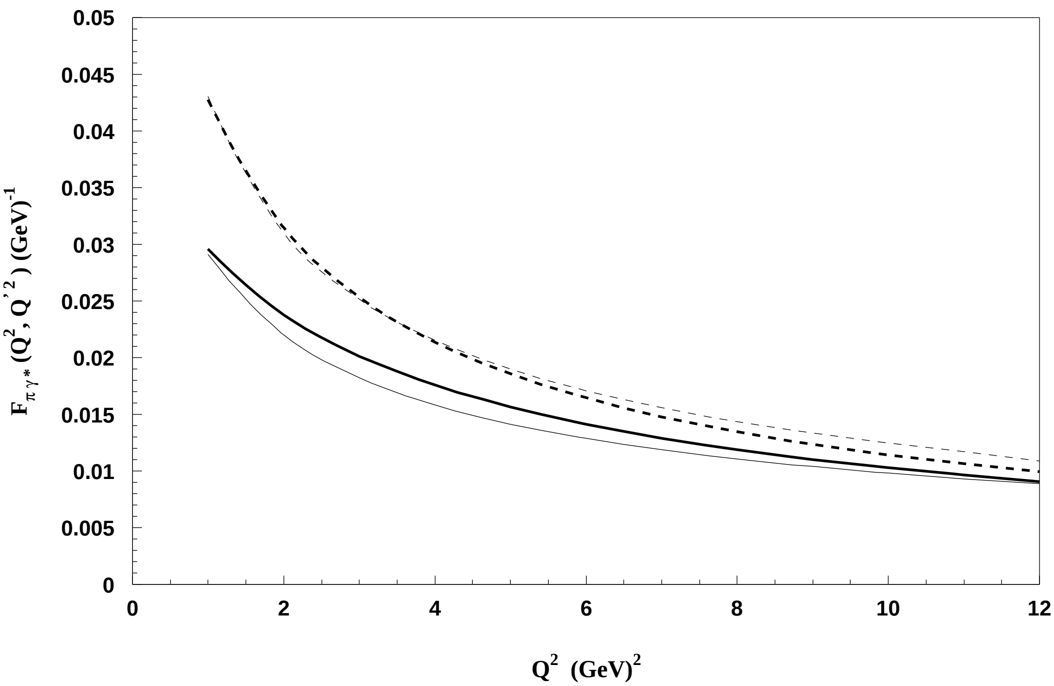


Fig. 1



0.3
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12
14
16
18
20
$Q^2 F_{\pi\gamma}(Q^2)$ (GeV)
Q^2 (GeV)^2

Fig. 2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0.04
0.035
0.03
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0.02
0.015
0.01
0.005
0
0
2
4
6
8
10
12
$F_{\pi\gamma^*}(Q^2, Q'^2)$ (GeV)$^{-1}$
Q^2 (GeV)^2

Fig. 3